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Dynamics of atom–cavity system and elimination of momentum recoil

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Abstract. In the framework of algebraic dynamics, we investigate an atom–cavity system with atomic centre-of-mass motion included. Using nonlinear transformations, the Hamiltonian is linearized in terms of Lie algebraic generators so that the algebraic dynamical structure of the system appears quite clear and its solution is easy to handle by an algebraic dynamical method. In more detail, for the three-level system of Ξ configuration, we show that the momentum recoil of the atom can be eliminated under certain conditions.

1. Introduction

Quantum-optical interactions of atoms with coherent fields may be studied at different levels of sophistication. In early works of quantum optics [1, 2] attention was simply concentrated on the internal atomic transitions and photon states without regarding the external motion of the atom. However, with the development of cavity quantum electrodynamic techniques, one needs to describe the system in a more precise standard, and the centre-of-mass recoil of the atom should be taken into account. Such a consideration is also crucial in studies of many fields such as the laser cooling of atoms [3, 4] and atomic interferometers [5, 6].

For the case of two-level atoms traversing an optical ring cavity, the exact solution could be achieved by using either a Laplace transformation or the transformation introduced in [7]. However, it is difficult to apply these two methods to more complex systems—the multi-level systems because of their nonlinearity. Alternatively, to circumvent this difficulty we shall employ an algebraic dynamical approach [8] to treat the multi-level systems. The key idea behind this method is to introduce a canonical transformation for the Hamiltonian and linearize it in terms of a set of Lie algebraic generators. Such a transformation can be viewed as the dressed state transformation which has been extensively employed in the investigation of atom–field interactions over the years [9, 11]. According to algebraic dynamics [8], linear algebraic dynamical systems are integrable and thus solvable, and their solutions are easy to handle. Then, by examining the time evolution properties of the system, we show that for a three-level Ξ -type system the recoil of the atom can be eliminated under certain conditions.

2. Dynamics of the system

2.1. The two-level case

To illustrate our method, we first consider the simplest situation: a two-level atom with an energy difference $\hbar\omega_0$ interacting with a one-mode electromagnetic cavity field which is quantized. In the rotating wave approximation the Hamiltonian is given by

$$H = \frac{\hat{p}_x^2}{2m} + \hbar\omega_0 S_z + \hbar\omega a^\dagger a + \hbar\Omega (a^\dagger S_- e^{-ikx} + a S_+ e^{ikx}) \quad (1)$$

where we take the propagating direction of the cavity field as the x -axis. a and a^\dagger denote the annihilation and creation operators of the quantized radiation field of the cavity with frequency ω , and k represents its wavenumber. Ω is the coupling constant between the atom and photons and is assumed real. Note that in this paper we consider an extended atom coupled to the electromagnetic field of a unidirectional ring cavity. The model Hamiltonian (1) is just an improved two-level JC model with the atomic centre-of-mass motion included and quantized as indicated by its kinetic energy operator (the first term in equation (1)). It is not difficult to see that if the atomic centre-of-mass motion is neglected ($k = 0$), the Hamiltonian (1) reduces to the naive JC model which is the simplest model in quantum electrodynamics. Of course, in the naive JC model, the cavity loss is neglected for simplicity, the model is thus ideal and only applicable to a cavity system with rather small loss. Inclusion of cavity loss will result in increased complexity of the model, and the dissipation due to the cavity loss is usually described by a non-Hermitian Hamiltonian which is difficult to deal with.

A straightforward analysis of the system shows that there are two invariants: one is the excitation number operator with the form $N = a^\dagger a + S_z + \frac{1}{2}$; the other is the total momentum of the system, $P_{\text{tot}} = \hat{p}_x + \hbar k a^\dagger a$. It is convenient to introduce the following notation:

$$P_g = P_{\text{tot}} - \hbar k N = \hat{p}_x - \hbar k (S_z + \frac{1}{2}). \quad (2)$$

P_g is also an invariant of the system which represents the atomic momentum of the ground state. From equation (2) it is clearly understood that the momentum exchange between the atom and photons is always connected with a change of atomic internal states, i.e., there is an interplay between internal and external atomic degrees of freedom. According to equation (2), we can express the kinetic energy in terms of the internal degree of freedom as follows:

$$\frac{\hat{p}_x^2}{2m} = \frac{1}{2m} \left[P_g + \hbar k \left(S_z + \frac{1}{2} \right) \right]^2 = \frac{1}{2m} \left(P_g^2 + \hbar k P_g + \frac{1}{2} \hbar^2 k^2 \right) + \frac{1}{2m} (\hbar^2 k^2 + 2\hbar k P_g) S_z. \quad (3)$$

Thus, the Hamiltonian can be rewritten as

$$H = E(N, P_g) + \Delta S_z + \hbar\Omega (a^\dagger S_- e^{-ikx} + a S_+ e^{ikx}) \quad (4)$$

where,

$$\begin{aligned} E(N, P_g) &= \frac{1}{2m} \left(P_g^2 + \hbar k P_g + \frac{1}{2} \hbar^2 k^2 \right) + \hbar\omega \left(N - \frac{1}{2} \right) \\ \Delta &= \Delta(P_g) = \frac{\hbar k P_g}{m} + \frac{\hbar^2 k^2}{2m} + \hbar\omega_0 - \hbar\omega. \end{aligned} \quad (5)$$

Knowing that the operators N and P_g are invariants, we can treat them as constants in their irreducible representations. Now one can verify that the Hamiltonian (4) can be linearized in

terms of $su(2)$ algebra generators $\{S_z, N^{-1/2}aS_+e^{ikx}, N^{-1/2}a^\dagger S_-e^{-ikx}\}$ which are nonlinear expressions and obey the following commutation relations:

$$\begin{aligned} [S_z, N^{-1/2}aS_+e^{ikx}] &= N^{-1/2}aS_+e^{ikx} \\ [S_z, N^{-1/2}a^\dagger S_-e^{-ikx}] &= -N^{-1/2}a^\dagger S_-e^{-ikx} \\ [N^{-1/2}aS_+e^{ikx}, N^{-1/2}a^\dagger S_-e^{-ikx}] &= 2S_z. \end{aligned} \quad (6)$$

To solve the system now becomes a simple task. One can use the following transformation (an $SU(2)$ group element) to diagonalize the Hamiltonian (4):

$$U_g = \exp \left[\frac{\theta}{N^{1/2}} (aS_+e^{ikx} - a^\dagger S_-e^{-ikx}) \right] \quad (7)$$

where

$$\theta = \theta(N, P_g) = -\arctan \frac{\sqrt{\frac{\Delta^2}{4} + \hbar^2 \Omega^2 N} - \frac{\Delta}{2}}{\hbar \Omega N^{1/2}}. \quad (8)$$

In addition, the dressed Hamiltonian is obtained as

$$H' = U_g^{-1} H U_g = E(N, P_g) + \sqrt{\Delta^2 + 4\hbar^2 \Omega^2 N} S_z. \quad (9)$$

From the above treatment one can find that the new expression for the Hamiltonian in terms of Lie algebraic generators has clearly manifested its $su(2)$ algebraic dynamical structure and its solution is easy to handle by the algebraic dynamical method. In the following section we shall show that the analogous description is also applicable to a more complex system.

2.2. Three-level case

Now we apply the algebraic dynamical method to the three-level system of Ξ configuration, which is interesting, important, and not treated so far to our knowledge. The three levels of the atom are denoted as $|1\rangle$, $|2\rangle$, and $|3\rangle$ with corresponding energies E_i ($i = 1, 2, 3$) respectively. Assume $E_3 > E_2 > E_1$, and the dipole-induced transitions $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ are mediated by photons of two different modes of cavity fields, which are characterized by the photon operators a_1 and a_2 with corresponding frequencies ω_1 and ω_2 . As the centre-of-mass motion of the atom is included, the system is described by the Hamiltonian

$$\begin{aligned} H = \frac{\mathbf{p}^2}{2m} + \sum_{i=1}^3 E_i \sigma_{ii} + \hbar \omega_1 a_1^\dagger a_1 + \hbar \omega_2 a_2^\dagger a_2 \\ + \hbar g_1 (\sigma_{21} a_1 e^{i\mathbf{k}_1 \cdot \mathbf{r}} + \text{h.c.}) + \hbar g_2 (\sigma_{32} a_2 e^{i\mathbf{k}_2 \cdot \mathbf{r}} + \text{h.c.}) \end{aligned} \quad (10)$$

where coupling constants g_1 and g_2 are assumed to be real, $\sigma_{ij} = |i\rangle\langle j|$ are atomic level transition operators, and \mathbf{k}_1 and \mathbf{k}_2 are wavevectors of the two cavity modes. Based on algebraic dynamical analysis similar to that in the above section, we obtain three invariants of the system:

$$N_1 = a_1^\dagger a_1 + 1 - \sigma_{11} \quad N_2 = a_2^\dagger a_2 + \sigma_{33} \quad (11)$$

$$\mathbf{P}_i = \mathbf{P}_{\text{tot}} - \hbar \mathbf{k}_1 (N_1 - 1) - \hbar \mathbf{k}_2 N_2 = \mathbf{p} + \hbar \mathbf{k}_1 \sigma_{11} - \hbar \mathbf{k}_2 \sigma_{33}. \quad (12)$$

Here, N_1 and N_2 are excitation number operators of the two modes and $\mathbf{P}_{\text{tot}} = \mathbf{p} + \hbar \mathbf{k}_1 a_1^\dagger a_1 + \hbar \mathbf{k}_2 a_2^\dagger a_2$ is the total momentum of the system. \mathbf{P}_i is the atomic momentum connected with the intermediate level $|2\rangle$. The kinetic energy operator of the atom can now be expressed as

$$\begin{aligned} \frac{\mathbf{p}^2}{2m} &= \frac{1}{2m} (\mathbf{P}_i - \hbar \mathbf{k}_1 \sigma_{11} + \hbar \mathbf{k}_2 \sigma_{33})^2 \\ &= \frac{1}{2m} [\mathbf{P}_i^2 + (\hbar^2 \mathbf{k}_1^2 - 2\hbar \mathbf{P}_i \cdot \mathbf{k}_1) \sigma_{11} + (\hbar^2 \mathbf{k}_2^2 + 2\hbar \mathbf{P}_i \cdot \mathbf{k}_2) \sigma_{33}]. \end{aligned} \quad (13)$$

Introducing the following generators by nonlinear transformations

$$\begin{aligned} A_{12}^r &= N_1^{-1/2} \sigma_{12} a_1^\dagger e^{-ik_1 \cdot r} & A_{23}^r &= N_2^{-1/2} \sigma_{23} a_2^\dagger e^{-ik_2 \cdot r} \\ A_{13}^r &= A_{12}^r A_{23}^r = N_1^{-1/2} N_2^{-1/2} \sigma_{13} a_1^\dagger a_2^\dagger e^{-i(k_1+k_2) \cdot r} \\ A_{ii}^r &= \sigma_{ii} & (A_{ij}^r)^\dagger &= A_{ji}^r \quad (i, j = 1, 2, 3) \end{aligned} \quad (14)$$

one can verify that the above operators span an $su(3)$ algebra and, especially in the irreducible eigenspace of the operators N_1 , N_2 and P_i , they form an associative algebra isomorphic to that spanned by the operators σ_{ij} . It is easily shown that the following project property exists:

$$A_{ij}^r A_{kl}^r = \delta_{jk} A_{il}^r \quad \sum_{i=1}^3 A_{ii}^r = 1. \quad (15)$$

We can now write the Hamiltonian (10) in terms of the operators A_{ij}^r :

$$\begin{aligned} H &= E_0(N_1, N_2, \mathbf{P}_i) + \Delta_1(\mathbf{P}_i) A_{11}^r + \Delta_3(\mathbf{P}_i) A_{33}^r \\ &\quad + \tilde{g}_1(N_1)(A_{21}^r + A_{12}^r) + \tilde{g}_2(N_2)(A_{23}^r + A_{32}^r) \end{aligned} \quad (16)$$

where

$$\begin{aligned} E_0(N_1, N_2, \mathbf{P}_i) &= E_2 + \hbar\omega_1(N_1 - 1) + \hbar\omega_2 N_2 + \frac{\mathbf{P}_i^2}{2m} \\ \Delta_1(\mathbf{P}_i) &= \frac{\hbar^2 \mathbf{k}_1^2}{2m} - \frac{\hbar \mathbf{P}_i \cdot \mathbf{k}_1}{m} - (E_2 - E_1 - \hbar\omega_1) \\ \Delta_3(\mathbf{P}_i) &= \frac{\hbar^2 \mathbf{k}_2^2}{2m} + \frac{\hbar \mathbf{P}_i \cdot \mathbf{k}_2}{m} + E_3 - E_2 - \hbar\omega_2 \\ \tilde{g}_1(N_1) &= \hbar\sqrt{N_1} g_1 & \tilde{g}_2(N_2) &= \hbar\sqrt{N_2} g_2. \end{aligned} \quad (17)$$

Since the operators N_1 , N_2 , and \mathbf{P}_i can be treated as constants in their common eigenspace, the Hamiltonian (16) is thus a linearized version of the Hamiltonian (10) in terms of the $su(3)$ generators A_{ij}^r , which is completely equivalent to equation (10) without any approximation.

3. Elimination of recoil effect

Generally, it is certain that atomic transition between two levels will cause a momentum transfer of photons to the atom (recoil effect). However, for a two-photon process of the three-level system, there might be a way for the recoil effect to be eliminated. The scheme can be designed as follows: the two cavity modes must have opposite propagating directions and identical frequency (of course, their polarization directions are different since they couple different transitions of the atom), so the two kinds of photons shall possess equal and opposite momentum; and as doing so, the detunings of the two transitions $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ must be large enough so that the middle level $|2\rangle$ can be adiabatically eliminated [10] and become a virtual state. As a result, the atom will transit between the levels $|1\rangle$ and $|3\rangle$ without changing its momentum.

Now let us give it a detailed calculation. As mentioned above, to carry out such a two-photon resonance process, we must tune the wavenumbers of the two cavity modes as $\mathbf{k}_2 = -\mathbf{k}_1 \equiv k_1 \hat{e}_x$ and make the cavity frequencies consistent with two-photon energy conservation: $E_3 - E_1 = \hbar\omega_1 + \hbar\omega_2 = 2\hbar\omega_1$. These two conditions straightforwardly cause the coefficients of the Hamiltonian (16) $\Delta_1(\mathbf{P}_i) = \Delta_3(\mathbf{P}_i) \equiv \Delta(\mathbf{P}_i)$ (here,

$P_i = P_i \hat{e}_x$). Now the Hamiltonian can be diagonalized by introducing the canonical transformation [11]

$$U_g = \exp[\alpha(A_{13}^r - A_{31}^r)] \exp[\beta(A_{23}^r - A_{32}^r)] \quad (18)$$

with

$$\alpha = \arctan \frac{\tilde{g}_1}{\tilde{g}_2} \quad \beta = \frac{1}{2} \arctan \frac{2\sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}}{\Delta}. \quad (19)$$

And the transformed Hamiltonian is obtained as

$$H' = U_g^{-1} H U_g = E_0 + \Delta A_{11}^r + \lambda_2 A_{22}^r + \lambda_3 A_{33}^r \quad (20)$$

where,

$$\lambda_{2,3} = \lambda_{2,3}(N_1, N_2, P_i) = \frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 + \tilde{g}_1^2 + \tilde{g}_2^2}. \quad (21)$$

Here '+' is for λ_2 and '-' for λ_3 . Noting the relation of (15), the evolution operator of the system can be calculated straightforwardly (setting $\hbar = 1$):

$$U(t) = e^{-iHt} = U_g e^{-iH't} U_g^{-1} = \sum_{i,j=1}^3 U_{ij}(N_1, N_2, P_i) A_{ij}^r \quad (22)$$

where,

$$\begin{aligned} U_{11}(N_1, N_2, P_i) &= e^{-iE_0 t} (\cos^2 \alpha e^{-i\Delta t} + \sin^2 \alpha \sin^2 \beta e^{-i\lambda_2 t} + \sin^2 \alpha \cos^2 \beta e^{-i\lambda_3 t}) \\ U_{22}(N_1, N_2, P_i) &= e^{-iE_0 t} (\cos^2 \beta e^{-i\lambda_2 t} + \sin^2 \beta e^{-i\lambda_3 t}) \\ U_{33}(N_1, N_2, P_i) &= e^{-iE_0 t} (\sin^2 \alpha e^{-i\Delta t} + \cos^2 \alpha \sin^2 \beta e^{-i\lambda_2 t} + \cos^2 \alpha \cos^2 \beta e^{-i\lambda_3 t}) \\ U_{12}(N_1, N_2, P_i) &= e^{-iE_0 t} \sin \alpha \sin \beta \cos \beta (e^{-i\lambda_3 t} - e^{-i\lambda_2 t}) \\ U_{23}(N_1, N_2, P_i) &= e^{-iE_0 t} \cos \alpha \sin \beta \cos \beta (e^{-i\lambda_3 t} - e^{-i\lambda_2 t}) \\ U_{13}(N_1, N_2, P_i) &= e^{-iE_0 t} \sin \alpha \cos \alpha (\sin^2 \beta e^{-i\lambda_2 t} + \cos^2 \beta e^{-i\lambda_3 t} - e^{-i\Delta t}) \\ U_{ij}(N_1, N_2, P_i) &= U_{ji}(N_1, N_2, P_i) \quad (i, j = 1, 2, 3). \end{aligned} \quad (23)$$

Assume that at $t = 0$ the internal state of the atom is in the ground state, and the density operator of the system can be factorized into three parts:

$$\rho(0) = |\Psi_F(0)\rangle \langle \Psi_F(0)| \otimes |\Psi_{p_x}(0)\rangle \langle \Psi_{p_x}(0)| \otimes |1\rangle \langle 1| \quad (24)$$

where

$$|\Psi_F(0)\rangle = \sum_{n_1, n_2} F_{n_1, n_2} |n_1, n_2\rangle \quad (25)$$

$$|\Psi_{p_x}(0)\rangle = \int dp_x C(p_x) |p_x\rangle. \quad (26)$$

In order to explore the dynamics of the atom we introduce reduced density operators $\rho^A(t)$ which describe the internal state of the atom:

$$\begin{aligned} \rho^A(t) &= \int dp_x \sum_{n_1, n_2} \langle n_1, n_2, p_x | \rho(t) | n_1, n_2, p_x \rangle \\ &= \int dp_x \sum_{n_1, n_2} \langle n_1, n_2, p_x | U(t) \rho(0) U^\dagger(t) | n_1, n_2, p_x \rangle. \end{aligned} \quad (27)$$

Since $\rho_{11}^A + \rho_{22}^A + \rho_{33}^A = 1$, we merely need to investigate the two quantities to denote the internal state evolution of the atom: the population inversion $W(t) = \rho_{33}^A(t) - \rho_{11}^A(t)$ and $\rho_{22}^A(t)$. From equations (22), (24) and (27), it is straightforward to obtain

$$\begin{aligned} W(t) &= \int dp_x \sum_{n_1, n_2} |C(p_x - \hbar k_1 - \hbar k_2)|^2 |F_{n_1+1, n_2+1}|^2 |U_{13}(n_1+1, n_2+1, p_x - \hbar k_2)|^2 \\ &\quad - \int dp_x \sum_{n_1, n_2} |C(p_x)|^2 |F_{n_1, n_2}|^2 |U_{11}(n_1, n_2, p_x + \hbar k_1)|^2 \\ &= \int dp_x \sum_{n_1, n_2} |C(p_x)|^2 |F_{n_1, n_2}|^2 [|U_{13}(n_1, n_2, p_x + \hbar k_1)|^2 \\ &\quad - |U_{11}(n_1, n_2, p_x + \hbar k_1)|^2] \end{aligned} \quad (28)$$

$$\rho_{22}^A(t) = \int dp_x \sum_{n_1, n_2} |C(p_x)|^2 |F_{n_1, n_2}|^2 |U_{12}(n_1, n_2, p_x + \hbar k_1)|^2. \quad (29)$$

To investigate the recoil effect of the atom, we need to calculate the momentum-distribution function of the atom, $P(p_x, t)$, defined as

$$\begin{aligned} P(p_x, t) &= \rho_{p_x, p_x}^A(t) = \sum_{n_1, n_2} \sum_{i=1}^3 \langle n_1, n_2, p_x, i | \rho(t) | n_1, n_2, p_x, i \rangle \\ &= \sum_{n_1, n_2} |F_{n_1, n_2}|^2 \{ |C(p_x)|^2 |U_{11}(n_1, n_2, p_x + \hbar k_1)|^2 \\ &\quad + |C(p_x - \hbar k_1)|^2 |U_{12}(n_1, n_2, p_x)|^2 \\ &\quad + |C(p_x)|^2 |U_{13}(n_1, n_2, p_x + \hbar k_1)|^2 \}. \end{aligned} \quad (30)$$

As is shown, the above calculations are valid for an arbitrary detuning Δ . To eliminate the middle level $|2\rangle$, we shall consider the large detuning case, $\Delta^2 \gg \tilde{g}_1^2 + \tilde{g}_2^2$, namely, the case where the detuning energy is vastly larger than the normalized interaction strength $\tilde{g}_{1,2}$. Under this condition, the following approximate relation is valid: $\sqrt{(\frac{\Delta}{2})^2 + \tilde{g}_1^2 + \tilde{g}_2^2} - \frac{\Delta}{2} \simeq \frac{\tilde{g}_1^2 + \tilde{g}_2^2}{\Delta}$. Hence we have $\sin^2 \beta \simeq \frac{\tilde{g}_1^2 + \tilde{g}_2^2}{\Delta^2} \rightarrow 0$ and $\cos^2 \beta \rightarrow 1$. It should be noted that since $\tilde{g}_{1,2}$ are functions of $n_{1,2}$ respectively (see equations (11) and (17)), the above large detuning condition should be satisfied for all photon numbers n appearing in the initial photon number distribution (25). As the initial condition (25) is known, the large detuning condition can be checked by numerical calculation. Thus we obtain

$$\begin{aligned} |U_{12}(n_1, n_2, p_x)|^2 &\simeq 0 \\ |U_{13}(n_1, n_2, p_x + \hbar k_1)|^2 &\simeq \frac{4\tilde{g}_1^2 \tilde{g}_2^2}{(\tilde{g}_1^2 + \tilde{g}_2^2)^2} \sin^2 \left(\frac{1}{2} \lambda_2 t \right) \\ |U_{11}(n_1, n_2, p_x + \hbar k_1)|^2 &= 1 - |U_{12}(n_1, n_2, p_x)|^2 - |U_{13}(n_1, n_2, p_x + \hbar k_1)|^2 \\ &\simeq 1 - \frac{4\tilde{g}_1^2 \tilde{g}_2^2}{(\tilde{g}_1^2 + \tilde{g}_2^2)^2} \sin^2 \left(\frac{1}{2} \lambda_2 t \right). \end{aligned} \quad (31)$$

Now the two quantities expressed by equations (28) and (29) have the following approximate expressions:

$$\begin{aligned} \rho_{22}^A(t) &\simeq 0 \\ W(t) &\simeq \int dp_x \sum_{n_1, n_2} |C(p_x)|^2 |F_{n_1, n_2}|^2 \left[\frac{8\tilde{g}_1^2 \tilde{g}_2^2}{(\tilde{g}_1^2 + \tilde{g}_2^2)^2} \sin^2 \left(\frac{1}{2} \lambda_2 t \right) - 1 \right]. \end{aligned} \quad (32)$$

And the momentum distribution function $P(p_x, t)$ is given by

$$P(p_x, t) \simeq |C(p_x)|^2 \sum_{n_1, n_2} |F_{n_1, n_2}|^2 [|U_{11}(n_1, n_2, p_x + \hbar k_1)|^2 + |U_{13}(n_1, n_2, p_x + \hbar k_1)|^2] \simeq |C(p_x)|^2. \quad (33)$$

As is expected, the atomic internal level $|2\rangle$ is removed and the population oscillates between levels $|1\rangle$ and $|3\rangle$. In addition, the atomic momentum distribution $P(p_x, t)$ retains its initial value during the whole evolution. Experimentally, the momentum recoil effect can be measured by looking at the momentum distribution of the outgoing atoms: the fact that it remains in the initial distribution indicates elimination of the recoil effect.

4. Conclusion

In this paper, within the framework of algebraic dynamics, we have studied the quantum dynamics of an atom-cavity system as atomic motion is included. We have given a clear description of the dynamical algebraic structure of the system and obtained its analytical solution. Based on this, we have shown that the elimination of the momentum recoil effect of the atom is possible in a three-level system under certain conditions. The results obtained may be useful in the design of quantum optical experiments.

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